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RANDOM CURDS AS MATHEMATICAL MODELS OF FRACTAL RHYTHM IN ARCHITECTURE

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The author Carl Bovill has suggested and described a method for generating rhythm in architecture with the help of random curds, as they are the mathematical models of unpredictable and uneven groupings which he recognizes in natural shapes and in natural processes. He specified the rhythm generated in this way as the fractal rhythm. Random curds can be generated by a simple process of curdling, as suggested by B. Mandelbrot. This paper examines the way in which the choice of probability for every stage or level of the curdling process, and the number of stages in the procedure of curdling, affect the characteristics of the obtained fractal object as a potential mathematical model of rhythm in the design process. At the same time, this paper examines the characteristics of rhythm in architecture which determine whether the obtained fractal object will be accepted as an appropriate mathematical model of the observed rhythm.

Key words: ***Fractal Rhythm in Architecture; Random Curds; Curdling; Unpredictable and Uneven Grouping; Naturalness.***

INTRODUCTION

Numerous studies have shown that fractal objects which belong to the class of random fractals can be successfully used to describe, manipulate, and simulate a lot of natural shapes and processes (Avnir *et al.*, 1998; Feder, 1988; Mandelbrot, 1982; Peitgen, 2004; Voss, 1988). Random curds are fractal objects belonging to the class of random fractals, which, according to Bovill (1996:92), represents a 'disconnected set of points that has a clustered characteristic'. Bovill (1996) took random curds as the mathematical models of natural rhythm, or natural distribution, and described the procedure of transferring this rhythm from a model to the rhythm in architecture, thus achieving the variation in architectural compositions similar to random clustering of matter in nature, such as, for example, the clusters of stars and galaxies (Mandelbrot, 1982). The author defined the rhythm, or the random and uneven distribution of similar elements generated in this way, as the fractal rhythm, or fractal distribution.

A lot of authors have studied the potential of some geometric objects, such as planes and space curves and surfaces, and concepts, such as the concept of cellular automata (CA), to support the generative processes in the field of architectural and urban design (e.g. Petruševski *et al.*, 2009; 2010). This paper continues the studies in the same direction, and examines the possibility of using fractal geometry as a design tool in the exploration of architectural and urban forms.

Various studies have been conducted in environmental psychology in order to research the influence of natural and built environment on people (e.g., Hartig *et al.*, 2003; Kaplan and Kaplan, 1989; Kaplan, 1995; Purcell *et al.*, 2001; Parsons, 1991; Ulrich, 1993; Van den Berg *et al.*, 2007). They have revealed that people give a greater preference to natural features and that natural features in our environment have a favorable effect on the psychological and physiological condition of people. However, modern urban life has reduced the opportunity of exposure to natural features, which, as scientists assume, can have long-term negative consequences. According to Joye (2006; 2007), we can mitigate this negative trend, at least partially, if we use the shapes and principles of fractal geometry, as the 'geometry of nature' (Avnir *et al.*, 1998; Mandelbrot, 1982) in architectural and urban planning and design.

Random fractals, as well as deterministic fractals, are the objects of fractal geometry, resulting from the constructional procedures which are usually recursive. But the process of the construction of random fractals includes a component of randomness, so the algorithms for their construction are nondeterministic. Thus, with deterministic or exact fractals, the same pattern is repeated in new iterations or with every change of scale, so here we can talk about the exact self-similarity. On the other hand, when it comes to random fractals, the invariability through different scales is statistical; namely we don't have the exact, but the statistical self-similarity and because of it, every magnified section looks similar to, but not exactly the same as the global pattern from which it is extracted (Mandelbrot, 1982; Voss, 1988). According to Voss (1988), the important thing here is the fact that the aforementioned feature of the statistical self-similarity represents the central characteristic of fractals in nature.

Certain studies have proved that the thing responsible for the visual feature of naturalness, sometimes ascribed to certain fractal objects, is the very component of randomness which is included into the process of generating such objects. The authors Peitgen *et al.* (2004:425) incorporated the component of randomness into the process of generating a deterministic fractal set, also known as the Koch curve, expecting to get a 'realistic natural shape'. The obtained fractal curve had the same fractal dimension as the exact

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Koch curve, but, according to the authors: 'the visual appearance is drastically different; it looks much more like the outline of an island than the original snowflake curve' (*ibid.*:425). Similarly, comparing the exact Koch curve and a random Koch curve, Taylor and Sprott (2008:119) emphasized that the former was not sufficiently similar to natural shapes, because 'the exact repetition of patterns creates cleanliness rarely found in natural forms'. On the other hand, the authors claimed that, owing to a certain measure of randomness, that 'artificial look' could be avoided with the latter curd and a 'more natural-looking fractal' could be obtained. (*ibid.*:119) Another important thing is that in some studies of aesthetic preferences across fractal objects, this feature, the so-called *naturalness*, appears as a significant correlate of greater preferences (Richards, 2001). Also, Bovill (1996:6) observed that in its shapes and in the way it changes over time, nature shows a certain measure of randomness, i.e. the right measure of predictability and randomness, or 'the mixture of order and surprise'. So, according to him, using random curds, which already have the component of randomness or 'surprise' in themselves, as a design tool, the randomness could be included into the design process, which is one of the ways to ensure that buildings are in sympathy with their natural surroundings.

RHYTHM IN ARCHITECTURAL COMPOSITION

The elements, units, or motifs of an architectural composition which are identical or have similar features or a similar role in the composition, are responsible for the experience of the visual rhythm in an observer because of their multiple appearances in the same or modified form (Arnheim, 1974; Ching, 2007). When we mention the visual features of elements, we think of their size or position, as much as of the distinctive quality of their shapes and surfaces, e.g. the colour or hue, texture, transparency, etc. In Figure 1, in multiple occurrences of the observed element, which clearly represents a single element on the specific scale of observation or at a given level of organization (for example, a facade panel or a floor tile), characterized by specific visual features (e.g. colour or shape), the actual values of its visual features are either repeated, or modified (in other words, they can be either identical or more or less distinct from element to element), thus generating a specific visual rhythm.

Since the topic of this paper is how to generate rhythm in an architectural composition, as part of the design process, then the concept of the value of visual features here won't refer to any possible value that the mentioned feature can theoretically have, but only to the specific value determined in advance for a specific observed case of rhythm generation in an architectural composition (e.g. if we take the windows on a front wall as the elements whose multiple appearances create the visual rhythm, then the specific values of the majority of visual features, e.g. heights and widths, or sill elevations, will be determined depending on the function of the inner space, story height, window type, etc.).

RANDOM CURDS AS MODELS OF RHYTHM

Random curds can be produced by a simple process of *curdling* (Mandelbrot, 1982). It is a cascade process, which results in contraction and, according to Mandelbrot (1982:84), it originated from the attempt to 'mimic reality by purely geometric means'.

To create these fractals we need a 'grid or lattice, made of intervals, squares, or cubes, each divided into b^E subintervals, subsquares, or subcubes; b is the lattice base' (*ibid.*:210), where E equals 1, 2 or 3, for intervals, squares, or cubes, respectively. Curdling, or random clustering is achieved by a sequence of binary random choices which decides the later fate of each of b^E subintervals, subsquares, or subcubes. With the 'probability $p<1$, the subinterval 'survives' as part of a precurd; otherwise, it dies off' (*ibid.*:211). With the 'surviving' intervals, squares, or cubes, we continue to the next stage of the cascade. By selecting the probability of the event for every stage of the process, as well as the number of stages, we can get a model of random clustering of elements, with desired characteristics. The probabilities $p=1/2$, $1/3$, $2/3$, etc., may be simply obtained by tossing a coin or a dice.

Number of Values of Visual Features

The values of a visual feature of an architectural element are determined in advance for each particular case of rhythm generation, in accordance with the specific role that the observed element plays in an architectural composition, or with other specific requirements. If the rhythm is generated with the help of random curds, then the number of determined values is very important.

Namely, if the number of determined values is two, one random curd can be the mathematical model of the rhythm, because a random curd on every scale of observation also consists of subintervals, subsquares, or subcubes, which can be in only one of these two conditions: 'survives' – 'does not survive', and the binary relation 'empty-full', or 'exists -does not exist' can be translated into 'this one exists - that one exists', so that each of the two possible conditions in the process of rhythm generation will be an analogue for one value of a visual feature of an architectural element. In Figure 2, we can see a random curd, generated in the process of curdling through two stages, with 4^2 subsquares in every stage. Each subsquare is in one of the two possible conditions.

If the number of determined values is greater than two, we suggest overlapping two or more random curds, provided they have the same size and number b^E of subintervals, subsquares, or subcubes.

If we overlap two random curds, then each of b^E subintervals, subsquares, or subcubes (on the given scale of observation) can occur in one of the four possible conditions: one curd 'survives', the other curd 'survives', both curds 'survive', neither curd 'survives'. Thus, by overlapping two

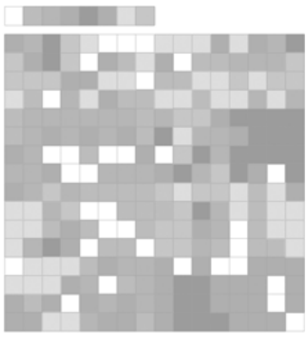


Figure 1. Multiple occurrences of elements in the same or modified form

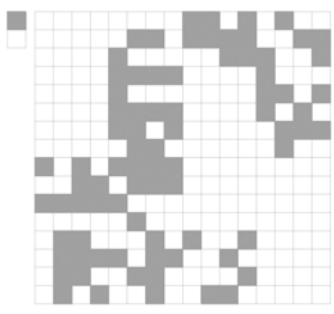


Figure 2. Mathematical model for rhythm of two values of visual features

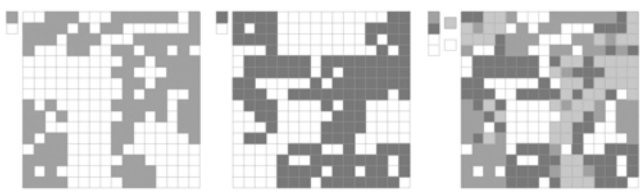


Figure 3. Mathematical model for rhythm of four values of visual features

random curds, we can obtain the mathematical model for those cases of the rhythm of the visual features of architectural elements where the number of the determined values is four. In Figure 3, we overlapped two random curds generated in the process of curdling through two stages, with 4^2 subsquares, each in every stage, and with the probability of 3/4 in the first stage and 2/3 in the second one.

Similarly, if we overlap three random curds, then each of b^E subintervals, subsquares, or subcubes (on a given scale of observation) occurs in one of the eight possible conditions. Thus, by overlapping three random curds, we can obtain the mathematical model for those cases of the rhythm of the visual features of architectural elements where the number of the determined values is eight. For example, the elements in eight colours in Figure 1 are arranged with the help of three overlapped random curds, generated in the process of curdling through two stages, with 4^2 subsquares, each in every stage, and with the probability of 3/4 in the first stage and 2/3 in the second one.

Therefore, the number of the determined values of visual features is important here, because by overlapping two or more random curds, we can get the mathematical models for rhythm generation in architectural compositions only for the cases where that number is equal to the number of all possible different outcomes for the overlapped random curds. Namely, if the number of overlapped random curds is marked with *n*, then the number of different possible outcomes, marked with R is equal to the sum of the number of combinations of *n* elements of the first class, second class, etc, to the *n*-th class, plus one (one refers to the outcome when all the overlapped subintervals, subsquares, or subcubes are those that 'don't survive'), which can be mathematically expressed as:

$$R = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = (1+1)^n = 2^n \quad (1)$$

Here, the subintervals, subsquares, or subcubes which 'do not survive' are treated the same, for all curds. Namely, they are treated as 'empty' or 'neutral', i.e. as such entities whose presence does not affect the outcomes.

When we use the mathematical model obtained by the described overlapping of two or more random curds as a design tool, the way we treat the overlapping of the subintervals, subsquares and subcubes which 'survive' is important for the final appearance of the generated rhythm. So, overlapping can cause 'the loss' of the initial value and the appearance of a new value, as shown in Figure 3, where the overlapping of two colours, blue and red for example, results in the third one - purple. In this case, the newly synthesized value is actually a sort of the median of the initial values and, as such, it is possible in the situations when the initial values are of the same kind, and mutually comparable on some scale of values.

Also, if we have, for example, some dimensional visual features, the values of which could be 'summed' in some way, this new value could possibly be the visual equivalent of their 'sum'. In Figure 4, which shows an irregularly perforated surface, we used three overlapped random curds as a mathematical model, generated in the process of curdling, through two stages, with 4^2 subsquares, each in every stage, and with the probability of 1/2 in both stages. Their subsquares which 'survived' were an analogue for the circular openings of the same size, but in three different positions in relation to the centre of the subsquare. The subsquares, on which two or three circular openings overlapped, were treated in such a way that the overlapped openings were replaced with the appropriate larger circular openings, as the visual equivalents of their 'sum'.

In addition to the described situation when we obtained a new value by overlapping the initial values (two colours gave the third one, or two smaller openings gave a larger one, etc.), which could be mathematically expressed as: a+b=c, there is also a possible approach where the initial values are not replaced with a new value, but retain their initial features even after overlapping, and remain visually present, which could be expressed as: a+b=ab.

First we overlapped two random curds, as shown in Figure 5a, and then three, as shown in Figure 5b, generated in the process of curdling through two stages, with 4^2 subsquares, each in every stage. Then we used the obtained model to

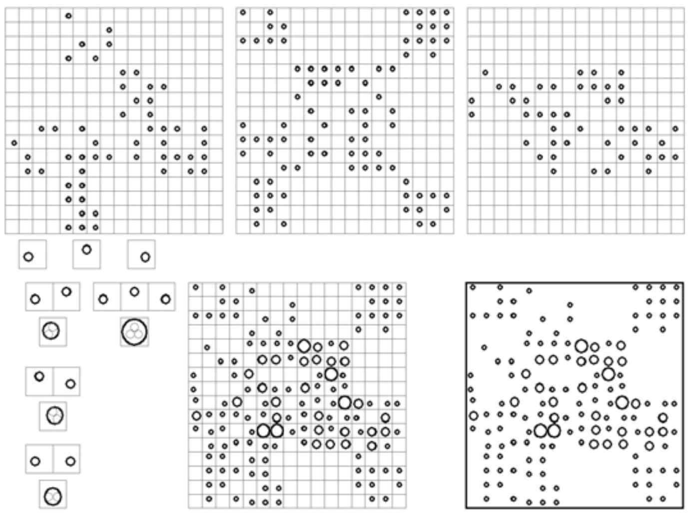
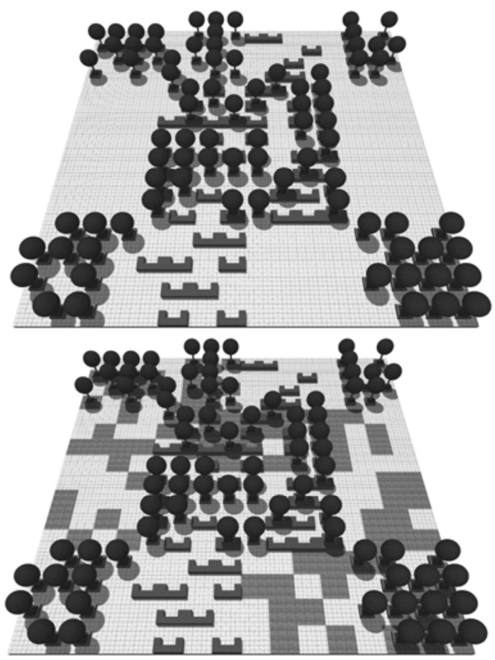


Figure 4. Perforated surface generated by means of three overlapped random curds



Figures 5a. and 5b. Distribution of the elements (trees, benches and floor tiles) of an imaginary park

distribute the elements of an imaginary park, so that the curds represented the analogues for the following elements: a tree, bench, or grey floor tiles. The 'empty' fields with all three curds were an analogue for white floor tiles.

Probability for Each Stage

The chosen probability for each stage of the generation process in the procedure of curdling is important because the presence or occurrence of the fields which 'survive' or 'do not survive' depends on it. Since we are talking about probability, the actual occurrence more or less approaches the expected occurrence. A different probability can be chosen for each stage of the process. For example, in Figure 3, we generated two random curds, with 4^2 subsquares, each in every stage, in the process of curdling through two stages, with the same probability of surviving: 3/4 in the first stage and 2/3 in the second one. The probability of a field to survive after the second stage was 3/4 x 2/3=1/2, which means that in both attempts, the expected number of the surviving fields was 256/2=128. The actual number of the surviving fields was 111 for the first curd, and 129 for the second one. It is important to emphasize that the possibility of prediction refers only to the number, but not to the distribution of the surviving fields, so the obtained curds with the same probability in the process and with the same number of stages can differ significantly in the visual sense, although they can have the same actual number of the surviving fields.

Also, when overlapping two or more random curds, the probability of possible outcomes depends on the individual probabilities for each overlapped random curd, and it equals the product of these individual probabilities. For example, in Figure 3, the probability of each of the four possible outcomes of the model obtained by overlapping two curds was 1/2 x 1/2=1/4.

The following question can be asked now: does the order of the 'phasic' probabilities somehow affect the characteristics of the obtained model, and thus the final appearance of the generated rhythm if we have the same number of stages in the process, the same 'phasic' probabilities which give the same final probability, and the same number of the expected surviving fields? In order to determine this, we generated two random curds, as shown in Figure 6, in the process of curdling through three stages, with the same phasic probabilities, but in a different order.

The order of the probabilities for the first curd was as follows: 3/4 for the first stage, 2/3 for the second one, and 1/3 for the third one. The probability that a subsquare would survive after the third stage was 3/4 x 2/3 x 1/3=1/6. Therefore, the expected number of the surviving subsquares was 4096/6=682. At the end of the process, 656 subsquares actually survived. The order of the probabilities for the second curd was as follows: 1/3 for the first stage, 2/3 for the second one, and 3/4 for the third one. The probability that a subsquare would survive after the third stage was the same as with the first curd; namely, it was 1/3 x 2/3 x 3/4=1/6. The expected number of the surviving subsquares was also 682. At the end of the process, 698 subsquares actually survived.

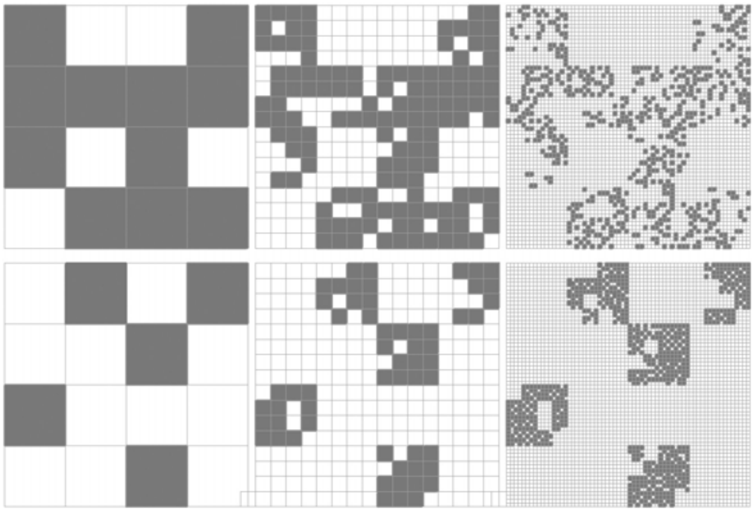


Figure 6. Two random curds generated with the same phasic probabilities, but in a different order

Comparing the order of probabilities for each stage of the generating process, and the final distribution of the surviving subsquares (and bearing in mind that the final amounts of the surviving subsquares with these curds were approximately the same), we could notice the following: that the first random curd, whose probabilities were reduced with every stage, looked 'sparser' compared to the other one, whose probabilities increased with every stage, because the total amount of the surviving fields was spread over a larger total surface even in the first stage. On the other hand, because of the preference for a smaller probability even in the first stage, the further procedure with the second random curd was limited to a smaller total surface, which later resulted in a higher concentration of the surviving subsquares, so the zones with the surviving fields looked denser or more compact.

At this point we can conclude that the characteristic of clustering will be more expressed in objects if we chose smaller probabilities of surviving in their initial stages, and here the first stage is especially significant in relation to subsequent stages. Because of this density, the boundaries between the surviving subsquares from the previous stage become more visible, so the visual presence of a geometric system in the process, i.e. the regular grid or lattice, is more emphasized, which can cause these curds and, further, the generated rhythm in architecture, to look less 'natural'.

Number of Stages

The number of stages in the process of curdling is important because the amount of the surviving subintervals, subsquares, or subcubes is reduced, compared to the initial amount, in every next stage of the generating process of this type of fractals. Namely, they cluster and group on a surface which gets smaller and smaller, whereas the distances between the surviving fields increase. Thus, the value of the fractal dimension decreases from one stage to another, and can be calculated using the 'box-counting method' for different stages of the process (Bovill, 1996; Mandelbrot, 1982). Also, the size of subintervals, subsquares, or subcubes decreases with every next stage, while their number increases at the same rate.

Here we can ask the next question regarding the use of these fractal objects as a design tool: can we reach a random curd, as a potential model of rhythm with the desired characteristics regarding the number and size of subintervals, subsquares, or subcubes, and regarding the desired relation of the two possible conditions 'survived – didn't survive', through a different number of stages of the process, and will the preference for the smaller or larger number of stages affect the characteristics of the obtained random curd, and even further, the features of the generated rhythm in architecture? For example, if we need a random curd as a mathematical model for paving a floor (dim. 20x20m) with tiles (dim. 30x30cm), in two colours, so that the presence of one colour is significantly smaller than the presence of the second colour (e.g. in the ratio of 1:8), then we can take a random curd with 64x64=4096 squares as a potentially appropriate model, where 1/9 of the squares should be the squares which survived all the previous stages. We can obtain such a model by generating a random curd in the process of curdling through three stages, where the probability that the subsquare will survive is 2/3 in the first stage, 1/2 in the second stage, and 1/3 in the third one. After the third stage, the probability will equal to the one we need, 2/3 x 1/2 x 1/3=1/9. In order to get 64 x 64 subsquares in the third stage, the grid, or raster should have 4^2 subsquares, each in every stage. However, the desired model can also be obtained by generating a random curd in the process of curdling through two stages, where the probability that the subsquare will survive is 1/3 in the first stage, and also 1/3 in the second one. After the second stage, the probability will also equal the one we need, 1/3 x 1/3=1/9. In order to get 64 x 64 subsquares in the second stage, the grid, or raster must have 8^2 subsquares, each in every stage.

In Figure 7, we can see two random curds, both of which had the same number and size of subsquares in the final stage, as well as approximately the same number of the surviving subsquares: the first one - 436, and the second one - 417 (the expected number of the surviving subsquares was 4096/9=455 for both of them).

Comparing these two curds, we can observe a difference in the way in which the surviving squares are clustered because of the different number of stages. Namely, in order to have the same probability of the surviving fields after the final stage, with the first curd (the one generated through three stages) the probabilities of surviving are greater for the first and the second stage, so the 'clustering of matter' is more gradual, and the 'surviving matter' looks sparser; it spreads over a larger basic surface. Whereas, for the same reason, in the second case (the curd generated through two stages), the probability in the first stage is smaller, and even as early as that, the clustering of matter, or the concentration of the further process on a smaller total surface, is greater. This connection between probability and concentration is described in the Section *Probability for Each Stage*.

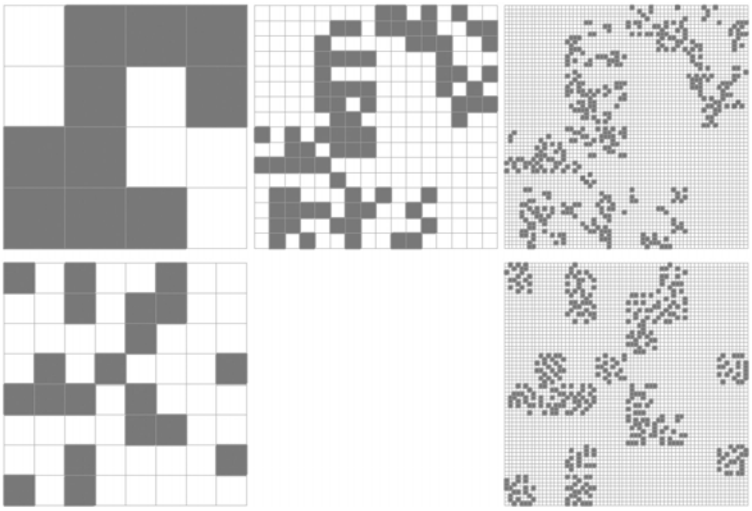


Figure 7. Two random curds generated through a different number of stages of the process

CONCLUSIONS

Random curds may be accepted as a design tool in architecture, with certain limitations. The limitations in the use of these fractal objects refer to the fact that they are generated in the process which is nondeterministic, so they have a component of randomness. Also, the process of curdling (the word derived from the verb *curdle*) (Mandelbrot, 1982) necessarily leads to an irregular distribution, unequal density, to a large concentration in certain sections, whereas some other sections remain completely 'empty'.

Certain characteristics of models, such as the size and number of subintervals, subsquares, or subcubes, can be determined in advance, in such a way that they completely correspond to the requirements of the program. Also, the relation between the presence of different elements, and the characteristics such as sparsity or density, or greater or smaller visibility of the regular grid or lattice, can be controlled and predicted to a certain extent, with the proper selection of the number of stages and probabilities for each stage. However, the exact position, or distribution of the surviving fields, cannot be predicted. It is only possible to repeat the process until we get the model whose distribution of the surviving fields, for example, best meets the requirements and suits the needs of a specific case. Also, sometimes only one segment of the generated curd can be taken as a model. In that case, it is possible to select the segment from the whole curd whose shape and the layout of the surviving fields would best suit the needs (Figure 8).

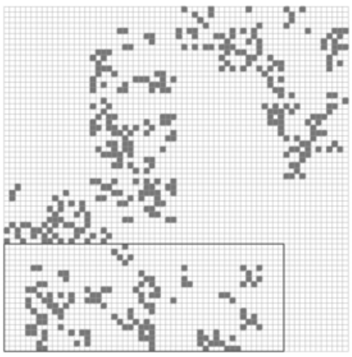


Figure 8. A segment of the generated curd with desirable shape and the layout of the surviving fields

Since the values of visual features of architectural elements are usually conditioned by a great number of requirements and limitations relating to different aspects of architecture (purpose, materialization, construction, social and natural context, etc.), (Ching, 2007), and on the other hand, since they often represent the groups of units which require the similar or identical treatment for the observed level of spatial organization, the following question can be asked: in what situations and to what extent can random curds be used as mathematical models in the design processes?

We will give only a general answer to this question here: because of the aforementioned random, unpredictable and uneven distribution of elements with the clustering tendency, random curds can be used as a design tool in the situations when the values of visual features are not strictly conditioned by various requirements and limitations placed before an architectural element (e.g. the layout of the floor tiles, or façade elements in several colours, as shown in Figure 1, where the issue of rhythm is sometimes reduced only to the visual aspect of the composition), or in the situations when the uneven distribution with the clustering tendency is not only acceptable, but also desirable. Such a case is shown in Figures 5a and 5b, where the component of randomness gives the distribution of the elements the wanted 'natural look', and on the other hand, the clustering of the elements (trees, benches, 'empty' fields) creates different microambients on the observed surface of a park, which can represent the framework for performing different activities, suit different users, and potentially satisfy different needs.

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